

MA 8109 SUMMARY LEC 4

Int. theory $\rightarrow$ probab. theory $\rightarrow$ stoch. proc. $\rightarrow$ stoch. int. $\rightarrow$ SDEs $\rightarrow \dots$

$(\Omega, \mathcal{F}, P)$ probability space $[P(\Omega) = 1]$

X random variable if $X: \Omega \rightarrow \mathbb{R}$ is $\mathcal{F}$ -meas.

Distribution/law of X : $\mu_X(B) = P(X^{-1}(B))$, $B \in \mathcal{B}$

$$E(g(X)) = \int_{\Omega} g(X) dP = \int_{\mathbb{R}} g(x) d\mu_X(x) \quad g: \mathbb{R} \rightarrow \mathbb{R} \text{ } \mathcal{B}\text{-meas.}$$

Characteristic func'n of X : $\Phi_X(u) = E(e^{iuX})$

X nd Gaussian $\Leftrightarrow \Phi_X(u) = e^{iu \cdot \mu - \frac{1}{2} u^T \Sigma u}$ for some $\mu \in \mathbb{R}^n$, symm. Σ

Independence

Events A, B independent if

$$P(A|B) = P(A) \quad \Leftrightarrow \quad P(A \cap B) = P(A) \cdot P(B)$$

Bayes law

$$P(A \cap B) = P(A|B) \cdot P(B)$$

Outcome $w \in B$ gives no info for whether $w \in A$